Efficient Iterative Decoding of LDPC in the Presence of Strong Phase Noise

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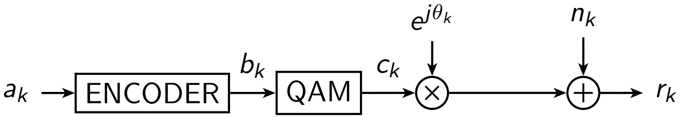
The 7th International Symposium on Turbo Codes & Iterative Information Processing

Gothenburg, Sweden
August 27, 2012

System Model



Phase Noise Equivalent Baseband Channel

$$r_k = c_k e^{j\theta_k} + n_k \,, \quad n_k \sim \mathcal{CN}(0, \sigma^2)$$

$$\theta_k = \theta_{k-1} + \Delta_k \,, \quad \Delta_k \sim \mathcal{N}(0, \sigma_\Delta^2)$$

Why is phase noise important?

Motivation

- Increase throughput in low end systems (low SNR)
- $\Rightarrow$
  Increase QAM constellation order
- $\Rightarrow$
  Increased sensitivity to phase noise

What can we do?

Use the code!

- LDPC can work well in **low SNR regions**
- Perform **iterative joint detection and estimation**

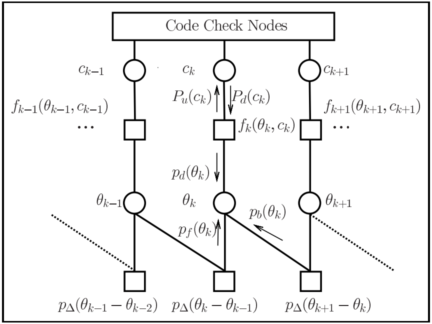
Goal

- Design a **low complexity** algorithm for providing **LLRs to the LDPC decoder**

Joint Detection & Estimation

The Factor Graph Approach,

$$p(\mathbf{c}, \boldsymbol{\theta}|\mathbf{r}) \propto p(\theta_0) \prod_{k=1}^{K-1} \underbrace{p(\theta_k|\theta_{k-1})}_{p_\Delta(\theta_k - \theta_{k-1})} \prod_{k=0}^{K-1} \underbrace{p(r_k|\theta_k, c_k)}_{f_k(c_k, \theta_k)} \mathbb{1}\{c_0^{K-1} \in \mathcal{C}\}$$



[From Barbieri, Colavolpe and Caire (2006)]

Sum and Product Algorithm

SPA Messages

- $p_f(\theta_k) = \int_0^{2\pi} p_f(\theta_{k-1}) p_d(\theta_{k-1}) p_\Delta(\theta_k - \theta_{k-1}) d\theta_{k-1}$
- $p_b(\theta_k) = \int_0^{2\pi} p_b(\theta_{k+1}) p_d(\theta_{k+1}) p_\Delta(\theta_{k+1} - \theta_k) d\theta_{k+1}$
- $p_d(\theta_k) = \sum_{m=0}^{M-1} P_d(c_k = e^{j\frac{2\pi m}{M}}) f_k(c_k, \theta_k)$
- $P_u(c_k) = \int_0^{2\pi} p_f(\theta_k) p_b(\theta_k) f_k(c_k, \theta_k) d\theta_k$

- Implementation problem - Phase messages are continuous!

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- One solution - Quantize the phase and perform approximated SPA

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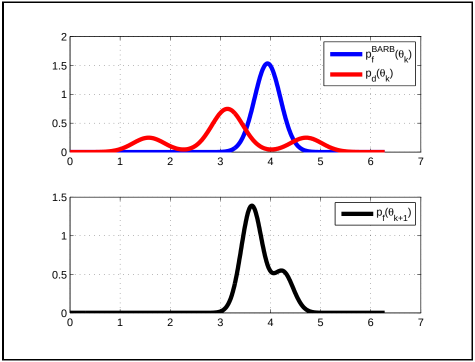
- Implementation problem - Phase messages are continuous!
- One solution - Quantize the phase and perform approximated SPA
- **Problem - High accuracy requires high complexity**

SPA Messages Approximation

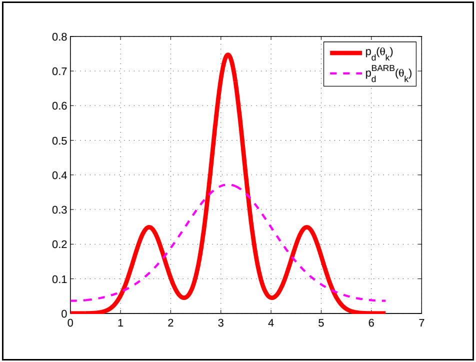
Barbieri, Colavolpe and Caire (2006) - **Single Tikhonov** canonical model

$$\underbrace{p_f(\theta_{k+1})}_{\text{Tikhonov Mixture}} = \int_0^{2\pi} \underbrace{p_f^{BARB}(\theta_k)}_{\text{Single Tikhonov}} \underbrace{p_d(\theta_k)}_{\text{Tikhonov Mixture}} \underbrace{p_\Delta(\theta_{k+1} - \theta_k)}_{\text{Gaussian}} d\theta_k$$



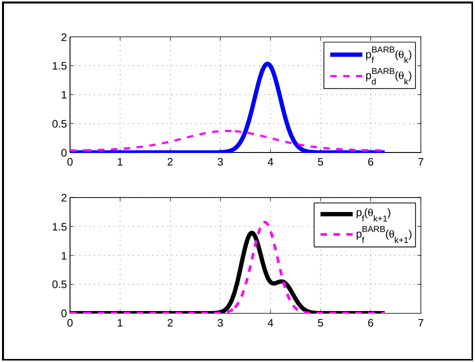
Approximating $p_d(\theta_k)$ as a Single Tikhonov

In order to approximate $p_f(\theta_k)$ as a single Tikhonov, Barbieri, et, al. suggested to approximate (Using Gaussian approximation) the $p_d(\theta_k)$ messages as a single Tikhonov - $p_d^{BARB}(\theta_k)$



Approximated Forward Recursion Equation

$$\underbrace{p_f^{BARB}(\theta_{k+1})}_{\text{Single Tikhonov}} = \int_0^{2\pi} \underbrace{p_f^{BARB}(\theta_k)}_{\text{Single Tikhonov}} \underbrace{p_d^{BARB}(\theta_k)}_{\text{Single Tikhonov}} \underbrace{p_\Delta(\theta_{k+1} - \theta_k)}_{\text{Gaussian}} d\theta_k$$



Question

Can we do better?

Answer

Yes!

Approximating $p_f(\theta_{k+1})$ as a Single Tikhonov

Instead of approximating the mixture $p_d(\theta_k)$, we will approximate the mixture $p_f(\theta_{k+1})$,

$$p_f^{Mod.1}(\theta_{k+1}) = \underbrace{\int_0^{2\pi} p_f^{Mod.1}(\theta_k) p_d(\theta_k) p_\Delta(\theta_{k+1} - \theta_k) d\theta_k}_{Approximate}$$

Problem

- We can't use the Gaussian approximation here
- We need to find out how to **optimally cluster a Tikhonov mixture to a single Tikhonov!**

CMVM - Circular Mean and Variance Matching

Theorem (Shayovitz & Raphaeli 2012)

Given a circular distribution $f(\theta)$, the parameters of the Tikhonov distribution $g(\theta)$ which satisfy,

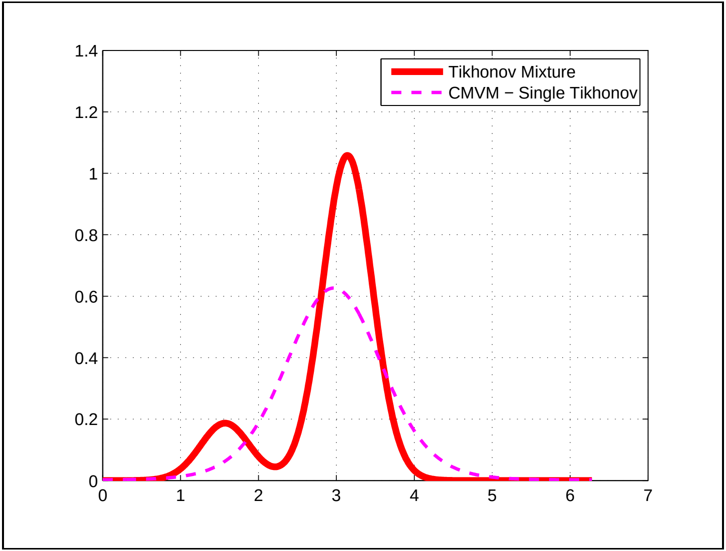
$$[\mu_{Circular}(g), \sigma^2_{Circular}(g)] = \arg\min_{\mu,\sigma^2} KL(f(\theta)||g(\theta))$$

Are given by:

$$\mu_{Circular}(g) = \mu_{Circular}(f)$$

$$\sigma^2_{Circular}(g) = \sigma^2_{Circular}(f)$$

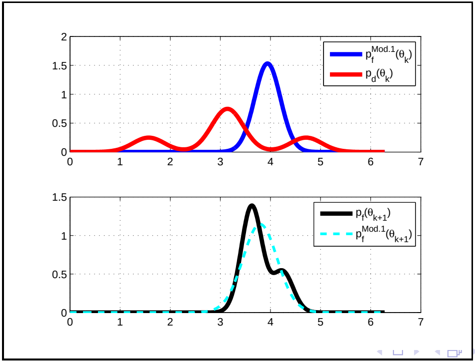
CMVM - Example



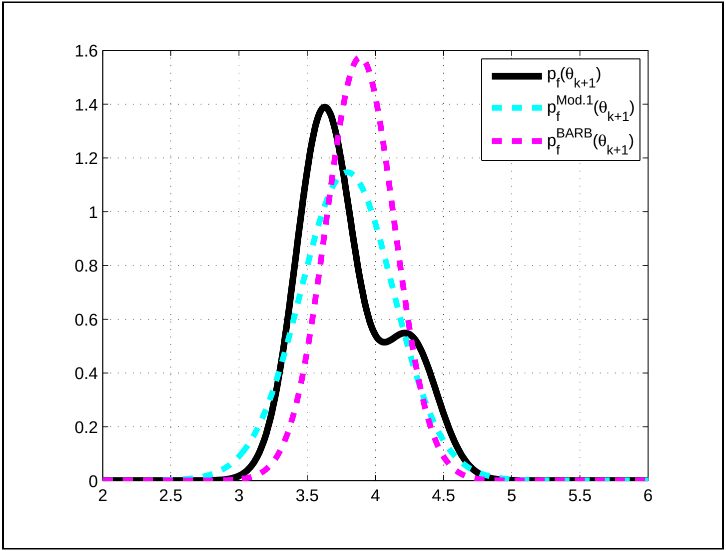
Approximating $p_f(\theta_{k+1})$ as a Single Tikhonov **Using CMVM**

We will approximate the mixture $p_f(\theta_{k+1})$ **using CMVM**,

$$p_f^{Mod.1}(\theta_{k+1}) = \underbrace{\int_0^{2\pi} p_f^{Mod.1}(\theta_k) p_d(\theta_k) p_\Delta(\theta_{k+1} - \theta_k) d\theta_k}_{Approximate}$$



Compare Approximations



Canonical Model No.1's Drawback

Problem

- In the first code iteration, the phase messages might be multi modal
- Canonical model No.1 is a **single Tikhonov** (one mode!)
- We might converge on only one mode
- $\Rightarrow$
  The estimation is vulnerable to **cycle slips** (phase ambiguities)

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Idea

- **Online approximation of probability of cycle slip event**
- Once this probability is high, use **pilot symbols to recover from cycle slips**

Approximating $p_f(\theta_k)$

We approximate the SPA messages using the following canonical model:

Model

$$p_f^{Mod.2}(\theta_k) = \alpha_k T_f(\theta_k) + (1 - \alpha_k)\frac{1}{2\pi}$$

- $T_f(\theta_k)$ is a single Tikhonov
- α_k approximates the **probability that a cycle slip hasn't occurred**

$p_f^{Mod.2}(\theta_{k+1}) = ?$

After insertion of canonical model no.2 to the forward recursion,

$$M(\theta_{k+1}) = \int_0^{2\pi} (\alpha_k T_f(\theta_k) + (1-\alpha_k)\frac{1}{2\pi}) p_d(\theta_k) p_\Delta(\theta_{k+1} - \theta_k) d\theta_k$$

Question

How to compute $T_f(\theta_{k+1})$?

$p_f^{Mod.2}(\theta_{k+1}) = ?$

After insertion of canonical model no.2 to the forward recursion,

$$M(\theta_{k+1}) = \int_0^{2\pi} (\alpha_k T_f(\theta_k) + (1-\alpha_k)\frac{1}{2\pi}) p_d(\theta_k) p_\Delta(\theta_{k+1} - \theta_k) d\theta_k$$

Question

How to compute $T_f(\theta_{k+1})$?

Answer

- Clustering all the modes $\Rightarrow$
  - Single mode approximation for multi modal messages
  - Convergence on a single mode $\Rightarrow$ cycle slips
- $\Rightarrow$
  Cycle slips may be avoided by selecting **only** the most probable modes and cluster them

How to choose the modes?

Selection & Clustering Algorithm

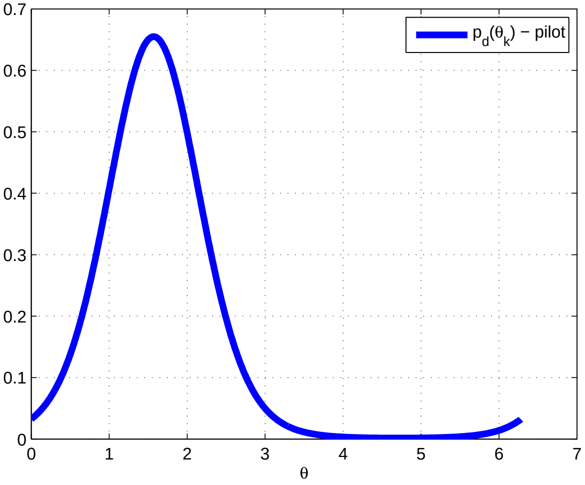
Given the mixture $M(\theta_{k+1})$,

1. Select the most probable mode
2. Find all the other modes similar to it
3. $T_f(\theta_{k+1}) \Leftarrow$ Cluster using CMVM all the selected modes
4. $\alpha_{k+1} \Leftarrow$ Sum up the modes' respective amplitudes

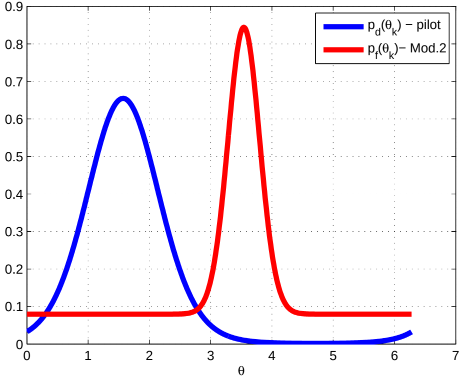
Intuition

This selection & clustering algorithm can be viewed as tracking a single phase trajectory while keeping a level of the likelihood of this trajectory.

The Forward Recursion when c_k is a Pilot

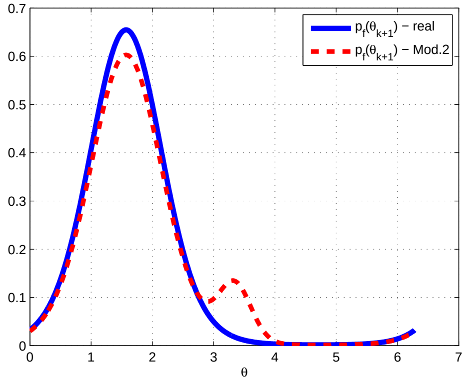


Canonical Model No.2

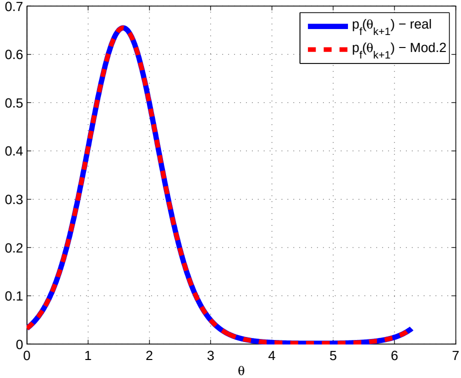


Before Selection & Clustering Algorithm

$$p_f(\theta_{k+1}) = \int_0^{2\pi} p_f(\theta_k) p_d(\theta_k) p_\Delta(\theta_{k+1} - \theta_k) d\theta_k$$



After Selection & Clustering Algorithm - Cycle Slip Recovered!



Complexity

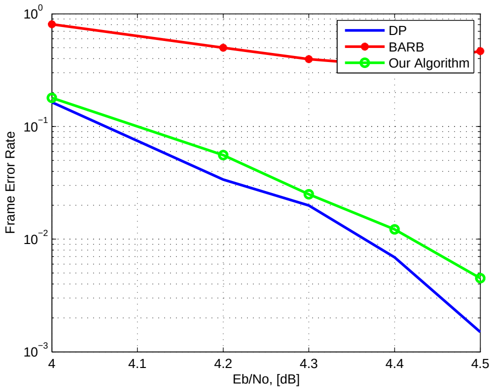
Computational load per code symbol per iteration for M-PSK constellation

	Quantized Phase (DP)	BARB	Model No.2
Operations	13ML+10QL-9L-3M	17M+11	22M+2
LUT	3ML+2QL-3L-M	3M+3	4M+1

M is the constellation order, L is the number of quantization levels and Q is a parameter for the DP algorithm

Simulation Results

A length 4608 LDPC code with rate 0.889
BPSK, $\sigma_\Delta = 0.1$[rads/symbol] and 1 pilot every 80 symbols
The quantized algorithm (DP) used 8 quantization levels.



Summary

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In this talk, we presented a new canonical model and tracking algorithm for **joint detection and estimation** of coded information in **strong phase noise** channels, with the following properties:

- Improved cycle slip robustness
- Low computational complexity
- Ability to work with high code rate & small number of pilots

Teaser - Tikhonov **Mixture** Canonical Model

First iteration messages may be multi modal ⇒ **mixture based canonical model**

Results for 8PSK and 0.05 rad per symbol