Multiple Hypotheses Iterative Decoding of LDPC in the Presence of Strong Phase Noise

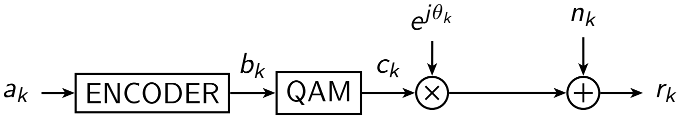
Shachar Shayovitz and Dan Raphaeli

Tel Aviv University

2012 IEEE 27-th Conenvtion of Electrical and Electronics Engineers in Israel

Eilat, Israel
November 16, 2012

System Model



Phase Noise Equivalent Baseband Channel

$$r_k = c_k e^{j\theta_k} + n_k \,, \qquad n_k \sim \mathcal{CN}(0, \sigma^2)$$

$$\theta_k = \theta_{k-1} + \Delta_k \,, \qquad \Delta_k \sim \mathcal{N}(0, \sigma_\Delta^2)$$

Why is phase noise important?

Motivation

- Increase throughput in low end systems (low SNR)
- $\Rightarrow$
  Increase QAM constellation order
- $\Rightarrow$
  Increased sensitivity to phase noise

What can we do?

Use the code!

- LDPC can work well in **low SNR regions**
- Perform **iterative joint detection and estimation**

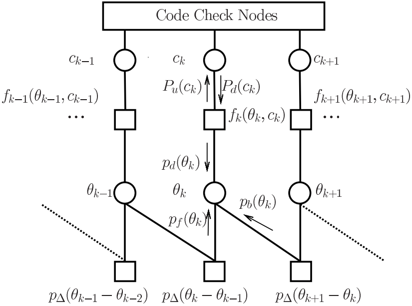
Goal

- Design a **low complexity** algorithm for providing **LLRs to the LDPC decoder**

Joint Detection & Estimation

The Factor Graph Approach,

$$p(\mathbf{c}, \boldsymbol{\theta}|\mathbf{r}) \propto p(\theta_0) \prod_{k=1}^{K-1} \underbrace{p(\theta_k|\theta_{k-1})}_{p_\Delta(\theta_k - \theta_{k-1})} \prod_{k=0}^{K-1} \underbrace{p(r_k|\theta_k, c_k)}_{f_k(c_k, \theta_k)} \mathbb{1}\{c_0^{K-1} \in \mathcal{C}\}$$



[From Barbieri, Colavolpe and Caire (2006)]

Sum and Product Algorithm

SPA Messages

- $p_f(\theta_k) = \int_0^{2\pi} p_f(\theta_{k-1}) p_d(\theta_{k-1}) p_\Delta(\theta_k - \theta_{k-1}) d\theta_{k-1}$
- $p_b(\theta_k) = \int_0^{2\pi} p_b(\theta_{k+1}) p_d(\theta_{k+1}) p_\Delta(\theta_{k+1} - \theta_k) d\theta_{k+1}$
- $p_d(\theta_k) = \sum_{m=0}^{M-1} P_d(c_k = e^{j\frac{2\pi m}{M}}) f_k(c_k, \theta_k)$
- $P_u(c_k) = \int_0^{2\pi} p_f(\theta_k) p_b(\theta_k) f_k(c_k, \theta_k) d\theta_k$

- Implementation problem - Phase messages are continuous!

Sum and Product Algorithm

SPA Messages

- $p_f(\theta_k) = \int_0^{2\pi} p_f(\theta_{k-1}) p_d(\theta_{k-1}) p_\Delta(\theta_k - \theta_{k-1}) d\theta_{k-1}$
- $p_b(\theta_k) = \int_0^{2\pi} p_b(\theta_{k+1}) p_d(\theta_{k+1}) p_\Delta(\theta_{k+1} - \theta_k) d\theta_{k+1}$
- $p_d(\theta_k) = \sum_{m=0}^{M-1} P_d(c_k = e^{j\frac{2\pi m}{M}}) f_k(c_k, \theta_k)$
- $P_u(c_k) = \int_0^{2\pi} p_f(\theta_k) p_b(\theta_k) f_k(c_k, \theta_k) d\theta_k$

- Implementation problem - Phase messages are continuous!
- One solution - Quantize the phase and perform approximated SPA

Sum and Product Algorithm

SPA Messages

- $p_f(\theta_k) = \int_0^{2\pi} p_f(\theta_{k-1})p_d(\theta_{k-1})p_\Delta(\theta_k - \theta_{k-1})d\theta_{k-1}$
- $p_b(\theta_k) = \int_0^{2\pi} p_b(\theta_{k+1})p_d(\theta_{k+1})p_\Delta(\theta_{k+1} - \theta_k)d\theta_{k+1}$
- $p_d(\theta_k) = \sum_{m=0}^{M-1} P_d(c_k = e^{j\frac{2\pi m}{M}})f_k(c_k, \theta_k)$
- $P_u(c_k) = \int_0^{2\pi} p_f(\theta_k)p_b(\theta_k)f_k(c_k, \theta_k)d\theta_k$

- Implementation problem - Phase messages are continuous!
- One solution - Quantize the phase and perform approximated SPA
- **Problem - High accuracy requires high complexity**

Canonical Model - Intro

- SPA messages are approximated using a family of distributions (finite parameters)
- Much lower computational complexity than quantization
- **Barbieri, Colavolpe and Caire (2006)** used a Single Tikhonov distribution **for all** SPA messages
- **Shayovitz and Raphaeli (2012)** used a **modified** Tikhonov distribution **only** for forward and backward messages

Canonical Model - Intro

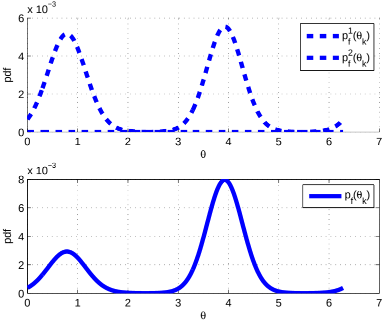
- SPA messages are approximated using a family of distributions (finite parameters)
- Much lower computational complexity than quantization
- **Barbieri, Colavolpe and Caire (2006)** used a single Tikhonov distribution **for all** SPA messages
- **Shayovitz and Raphaeli (2012)** used a **modified** Tikhonov distribution **only** for forward and backward messages

Problem

Single Tikhonov canonical models are not suitable for the first iteration in strong phase noise.

Canonical Model - Tikhonov Mixture

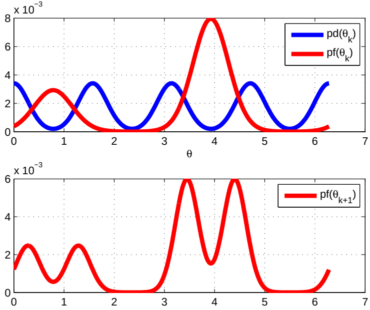
$$p_f(\theta_k) = \sum_{i=1}^{N_f} \alpha_i^f \frac{e^{Re[Z_i^f e^{-j\theta_k}]}}{2\pi I_0(|Z_i^f|)}$$



Problem

Mixture order grows exponentially!

$$\underbrace{p_f(\theta_{k+1})}_{\text{Tikhonov Mixture}} = \int_0^{2\pi} \underbrace{p_f(\theta_k)}_{\text{Tikhonov Mixture}} \underbrace{p_d(\theta_k)}_{\text{Tikhonov Mixture}} \underbrace{p_\Delta(\theta_{k+1} - \theta_k)}_{\text{Gaussian}} d\theta_k$$



Problem Formulation

Mixture Reduction

Given a Tikhonov mixture,

$$f(\theta) = \sum_{i=1}^{N} \alpha_i t_{f_i}(\theta)$$

Find a Tikhonov mixture with $M < N$

$$g(\theta) = \sum_{j=1}^{M} \beta_j t_{g_j}(\theta)$$

Which minimizes some distortion criterion,

$$D(f(\theta)||g(\theta))$$

Problem Formulation

- Kullback Leibler divergence is more natural for this setting than Integral square error (ISE)
- Optimal mixture reduction is NP hard
- Known mixture reduction algorithms such as: Salmond (1990),Williams & Maybeck (2003) and Runnalls (2006) don't work well

Why do those algorithms fail?

Fixed mixture order and **clustering errors** limit the performance:

- Small order will undergo cycle slips and create error floor
- Large order is too computationally demanding

New Problem Formulation - Dynamic Mixture Order

Objective

Given a Tikhonov mixture,

$$f(\theta) = \sum_{i=1}^{N} \alpha_i t_{f_i}(\theta)$$

Find the Tikhonov mixture $g(\theta)$ with the minimum number of components

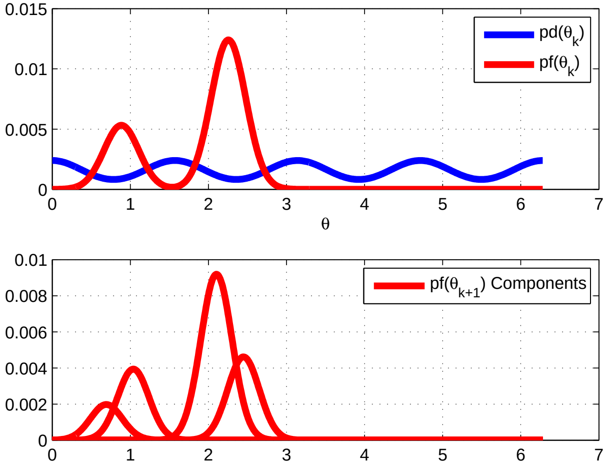
$$g(\theta) = \sum_{j=1}^{M} \beta_j t_{g_j}(\theta)$$

which satisfy,

$$D_{KL}(f(\theta)||g(\theta)) \leq \mu$$

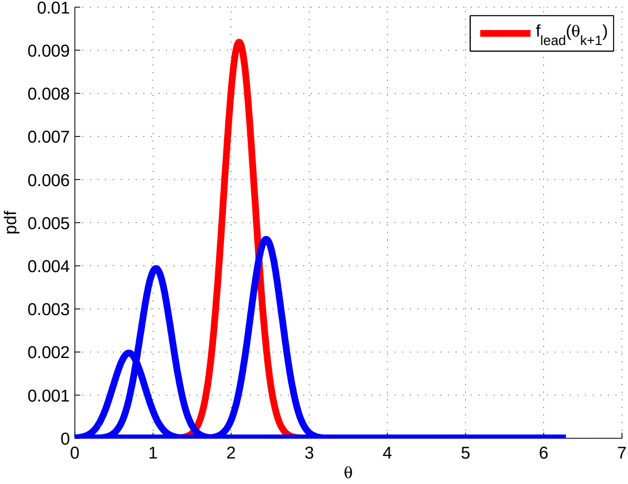
Mixture Reduction Algorithm

Suppose we need to reduce the dimensions of the following message $p_f(\theta_{k+1})$



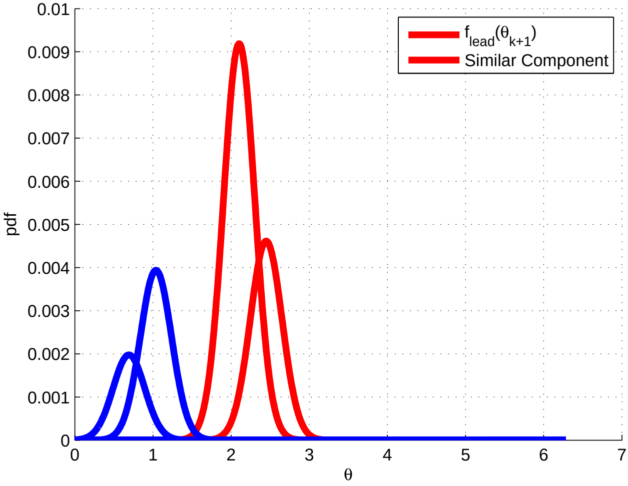
Mixture Reduction Algorithm

Choose the most probable mixture component and name it $f_{lead}(\theta_{k+1})$,



Mixture Reduction Algorithm

Find all other mixture components $f_i(\theta_{k+1})$ for which $D_{KL}(f_i(\theta_{k+1})||f_{lead}(\theta_{k+1})) \leq \mu$,



CMVM - Circular Mean and Variance Matching

Theorem (Shayovitz & Raphaeli 2012)

Given a circular distribution $f(\theta)$, the parameters of the Tikhonov distribution $g(\theta)$ which satisfy,

$$[\mu_{Circular}(g), \sigma^2_{Circular}(g)] = \arg\min_{\mu,\sigma^2} D_{KL}(f(\theta)||g(\theta))$$

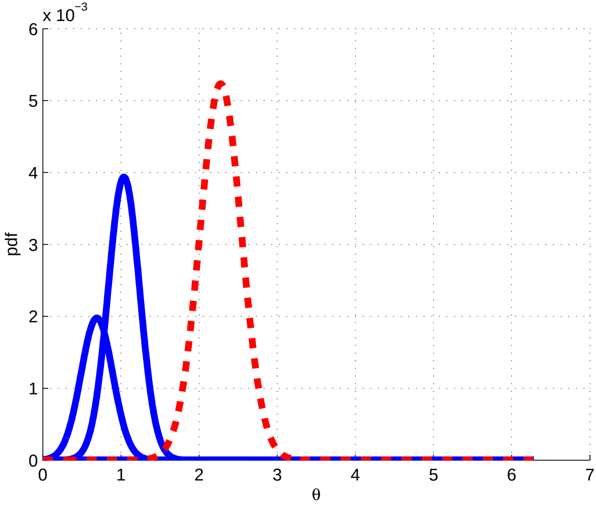
Are given by:

$$\mu_{Circular}(g) = \mu_{Circular}(f)$$

$$\sigma^2_{Circular}(g) = \sigma^2_{Circular}(f)$$

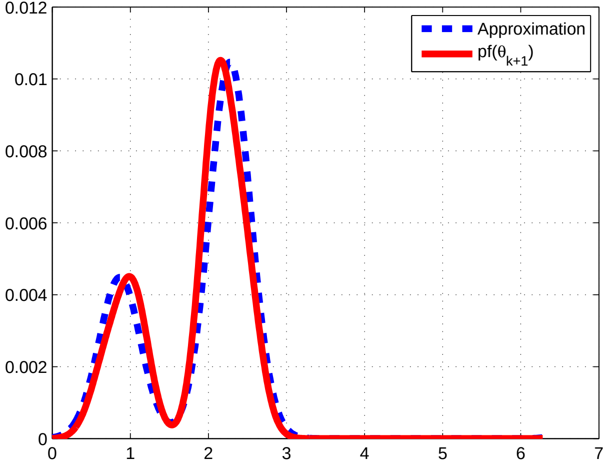
Mixture Reduction Algorithm

Cluster all the chosen mixture components using CMVM and get the first reduced mixture component $g_1(\theta_{k+1})$.



Mixture Reduction Algorithm

Eliminate the clustered components and iterate until there are no original mixture components left...

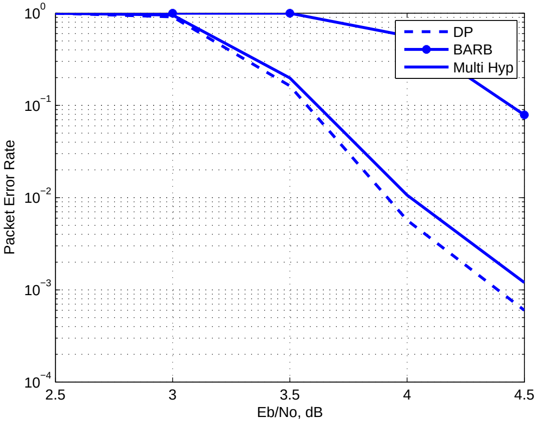


Algorithm Summary

- Low complexity
- For small μ, this algorithm guarantees that $D_{KL}(p_f(\theta_{k+1})||g(\theta_{k+1})) \leq \mu$
- Resulting average number of mixture components per symbol is low and decreases significantly with the LDPC iterations
- Very low probability of cycle slip

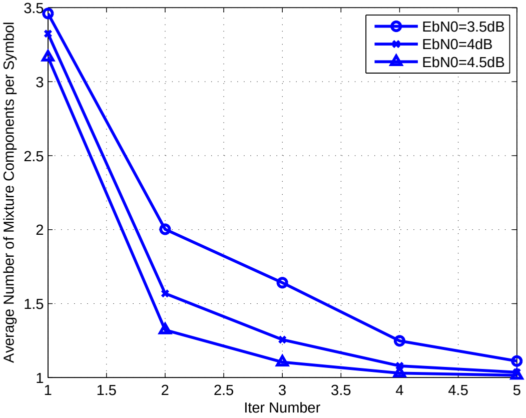
Simulation Results

- LDPC length 4608 with rate 0.75
- QPSK, $\sigma_\Delta = 0.1$[rads/symbol] and 1 pilot every 60 symbols
- The quantized algorithm (DP) used 8 quantization levels.



Average Mixture Order

We look at the average number of hypotheses per symbol as a measure of the complexity per iteration



Complexity

Computational load per code symbol per iteration for QPSK constellation

	DP	Multi Hyp, iter 1	Multi Hyp, iter 2
Operations	2324	403	115
LUT	476	133	45

M is the constellation order, L is the number of quantization levels and Q is a parameter for the DP algorithm

Summary

In this talk, we presented a **new approach** for **joint detection and estimation** of coded information in **strong phase noise** channels. We have presented a canonical model and mixture reduction algorithm with the following properties:

- Comparable PER to DP
- **Low computational complexity**
- Ability to work well with **high code rate**
- Ability to work well with small number of pilots or even **without any pilots**!
- Very low probability of cycle slip!

Backup - Mixture reduction algorithm

$j \leftarrow 1$
while $j \leq L$ or $|f(\theta)| > 0$ **do**
  $lead \leftarrow argmax_k\{\alpha_k\}$
  $idx \leftarrow lead$
  for $i = 1 \rightarrow |f(\theta)|$ **do**
    if $D_{KL}(f_i(\theta)||f_{lead}(\theta)) \leq \mu$ **then**
      $idx \leftarrow [idx, i]$
    end if
  end for
  $g_j(\theta) \leftarrow CMVM(\alpha(idx), f(idx))$
  $\beta_j \leftarrow \sum \alpha(idx)$
  $f(\theta) \leftarrow f(\theta) - \sum_{i \in idx} \alpha(i) f_i(\theta)$
  Normalize $f(\theta)$
  $j \leftarrow j + 1$
end while