Pilotless Communications Over Wiener Phase Noise Channels

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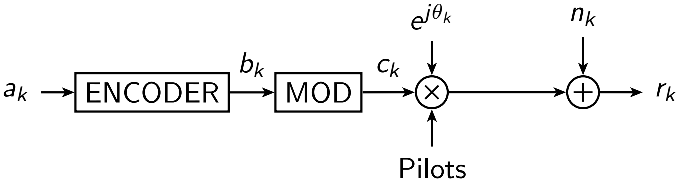
Phase Noise Channel Model



Phase Noise Equivalent Baseband Channel

$$r_k = c_k e^{j\theta_k} + n_k \,, \quad n_k \sim \mathcal{CN}(0, \sigma^2)$$

$$\theta_k = \theta_{k-1} + \Delta_k \,, \quad \Delta_k \sim \mathcal{N}(0, \sigma_\Delta^2)$$

Optimal Approach - Joint Detection & Estimation

Sum and Product Algorithm

- MAP detection -

$$\hat{b}_k = arg\max P(b_k|\mathbf{r})$$

- We will use the Sum and Product algorithm in order to compute

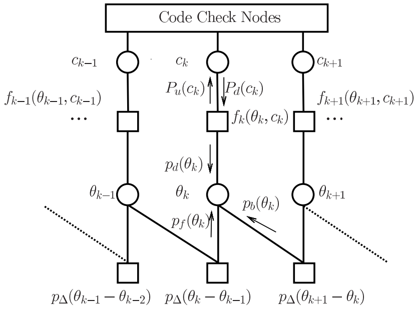
$$P(b_k|\mathbf{r})$$

- First we need to derive the factor graph for this problem
- Then, we can run the SPA message passing algorithm

Optimal Approach - Joint Detection & Estimation

The Factor Graph Approach,

$$p(\mathbf{c},\boldsymbol{\theta}|\mathbf{r}) \propto p(\theta_0)\prod_{k=1}^{K-1}\underbrace{p(\theta_k|\theta_{k-1})}_{p_\Delta(\theta_k-\theta_{k-1})}\prod_{k=0}^{K-1}\underbrace{p(r_k|\theta_k,c_k)}_{f_k(c_k,\theta_k)}\mathbb{1}\{c_0^{K-1}\in\mathcal{C}\}$$



[From Barbieri, Colavolpe and Caire (2006)]

Sum and Product Algorithm

SPA Messages

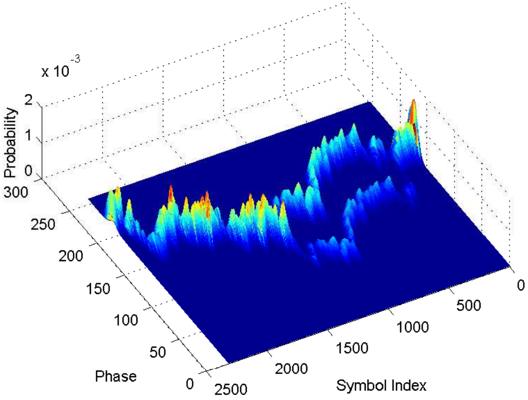
- $p_f(\theta_k) = \int_0^{2\pi} p_f(\theta_{k-1}) p_d(\theta_{k-1}) p_\Delta(\theta_k - \theta_{k-1}) d\theta_{k-1}$
- $p_b(\theta_k) = \int_0^{2\pi} p_b(\theta_{k+1}) p_d(\theta_{k+1}) p_\Delta(\theta_{k+1} - \theta_k) d\theta_{k+1}$
- $p_d(\theta_k) = \sum_{m=0}^{M-1} P_d(c_k) f_k(c_k, \theta_k)$
- $P_u(c_k) = \int_0^{2\pi} p_f(\theta_k) p_b(\theta_k) f_k(c_k, \theta_k) d\theta_k$

Phase Quantization

- θ_k is a continuous random variable
- SPA is approximated via phase quantization

Phase Noise Trajectories

Pilots kill the wrong trajectories!



Do we really need pilots?

Symmetrical Constellation

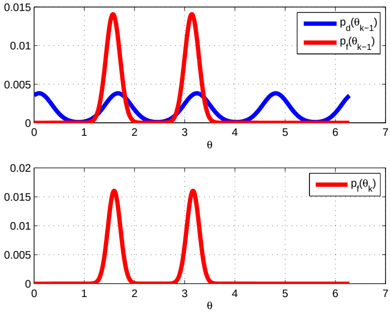
- Symmetrical constellations need pilots in order to bootstrap the estimation process (zero point in EXIT chart)
- Decoding symmetric constellations requires pilots due to inherent phase ambiguity (Zero point in EXIT charts)

Asymmetrical Constellations (AC)

Wrong phase trajectories will decay over time

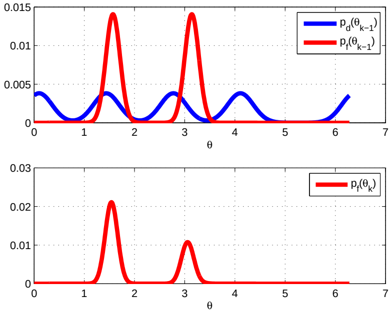
Symmetrical Constellation

$$\underbrace{p_f(\theta_k)}_{\text{Tikhonov Mixture}} = \int_0^{2\pi} \underbrace{p_f(\theta_{k-1})}_{\text{Tikhonov Mixture}} \underbrace{p_d(\theta_{k-1})}_{\text{Tikhonov Mixture}} \underbrace{p_\Delta(\theta_k - \theta_{k-1})}_{\text{Gaussian}} d\theta_{k-1}$$



Asymmetrical Constellation

$$\underbrace{p_f(\theta_k)}_{\text{Tikhonov Mixture}} = \int_0^{2\pi} \underbrace{p_f(\theta_{k-1})}_{\text{Tikhonov Mixture}} \underbrace{p_d(\theta_{k-1})}_{\text{Tikhonov Mixture}} \underbrace{p_\Delta(\theta_k - \theta_{k-1})}_{\text{Gaussian}} d\theta_{k-1}$$



Decay Factor - Definition

- A method to analyze the performance of AC in general
- SPA messages can be approximated using Tikhonov mixtures (Shayovitz & Raphaeli 2012)

$$p_f(\theta_k) = \sum_{i=1}^{N_f} \alpha_i^f \frac{e^{Re[Z_i^f e^{-j\theta_k}]}}{2\pi I_0(|Z_i^f|)}$$

- For asymmetrical constellations, the amplitude of the i^{th} "wrong" mixture component, α_i^k, is a decaying process
- The decay factor δ is defined as

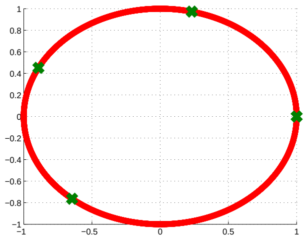
$$\delta = -\frac{\partial \log\left(\mathbb{E}(\alpha_i^k)\right)}{\partial k}$$

Skewed MPSK

Definition

$$s_m = e^{j\frac{2\pi m}{M+\epsilon}} \qquad m = 0, 1, ..., M-1. \tag{1}$$

where ϵ creates the skew in the constellation



Decay Factor - Skewed MPSK

Suppose the message $p_f(\theta_k)$ can be approximated using a Tikhonov mixture of order 2 (phase ambiguity ϕ at $k = 0$),

$$p_f(\theta_k) = \sum_{i=1}^{2} \alpha_i^k f_i^f(\theta_k) \tag{2}$$

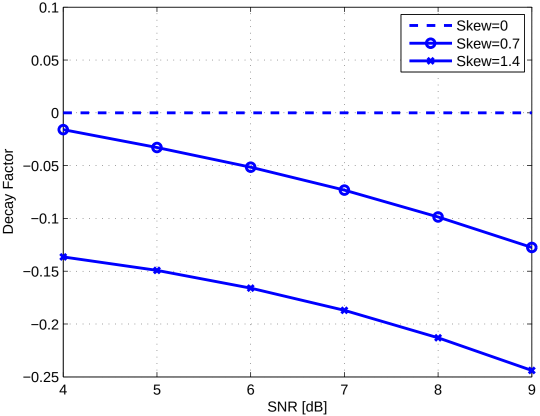
Then, the amplitude process can be described as,

$$\alpha_2^{k+1} = \frac{1}{1 + \frac{\alpha_1^0}{\alpha_2^0} \prod_{i=1}^{k} \frac{I_0(|\tilde{z}_1^{f,i}|)}{I_0(|\tilde{z}_2^{f,i}|)}} \tag{3}$$

Then the Decay Factor can be computed numerically using,

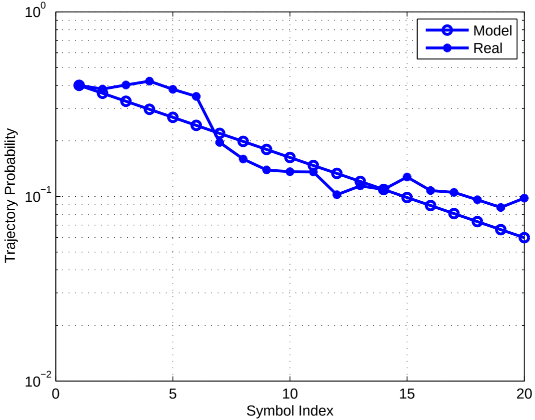
$$\delta = -\mathbb{E}\left[\log\left(\frac{I_0\left(\left|C + \frac{(c_k+n_k)e^{-j\frac{2\pi l}{M+skew}}}{\sigma^2}\right|\right)}{I_0\left(\left|Ce^{j\phi} + \frac{(c_k+n_k)e^{-j\frac{2\pi l}{M+skew}}}{\sigma^2}\right|\right)}\right)\right] \tag{4}$$

Skewed MPSK



As ϵ increases the decay factor is better

Decay Factor - Example

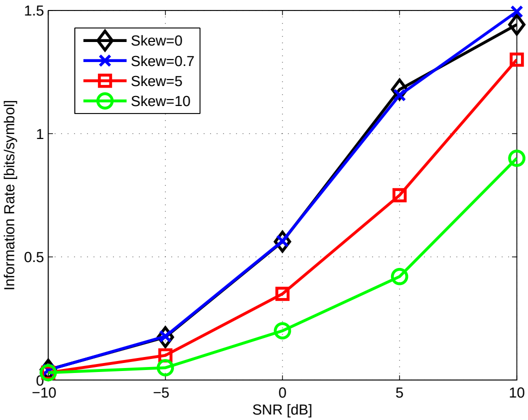


Question

For large ϵ, the minimum distance is increased (BER corrupted at High SNR), how does the skew impact the achievable information rate?

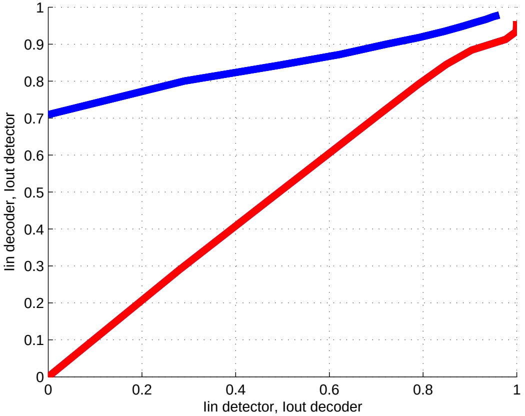
Skewed MPSK

Information Rates for different Skews



A skew of 0.7 seems to not lose IR, but can we use SPA without pilots?

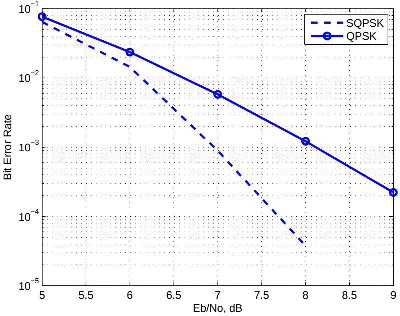
Skewed MPSK - EXIT Chart



For zero a priori LLR, we get an output of non zero LLR

Bit Error Rate Simulation Results

- 4096 length LDPC code with rate 0.89.
- SMPSK with a skew value of 0.7 without any pilots
- QPSK with one pilot every 40 symbols
- $\sigma_\Delta = 0.2$ [rads/symbol].



Summary

In this talk we have:

- proposed a new signal constellation for **pilotless** transmission of signals over Wiener phase noise channels.
- We also provided a method to analyze the performance of this constellation. This analysis can also be used to assess the performance of other signal constellations and provide insight.